Train track splitting sequences on the twice punctured torus

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The slides can be found on
https://jeanbellynck.github.io/

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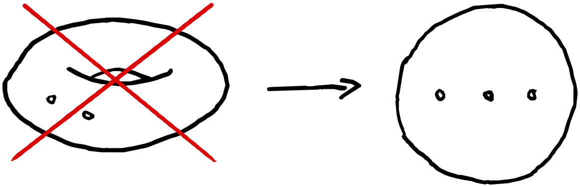
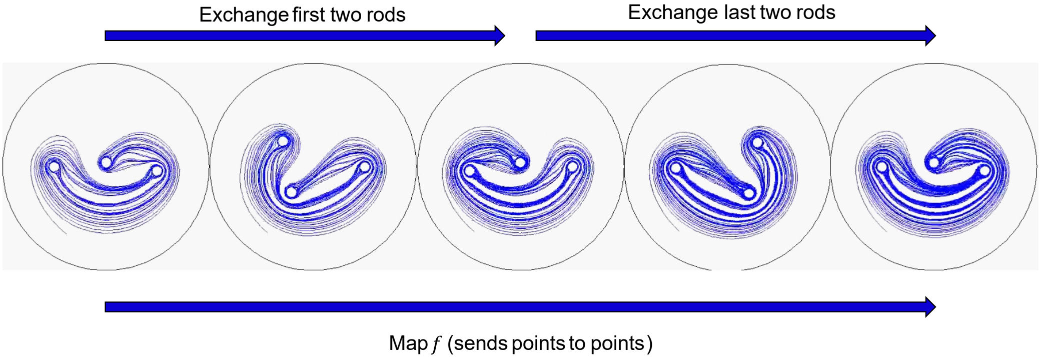
Disclaimer

For this presentation I will present the basics of my research using the more intuitive 3-punctured disc instead of the twice-punctured torus.



Fluid mixing

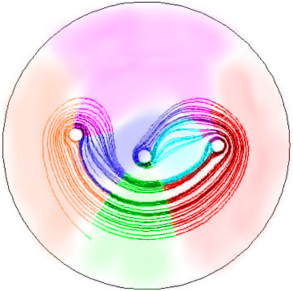
We add colouring into a round container with viscous fluid and repeatedly mix it using three rods as seen below.



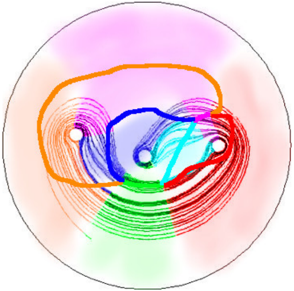
Quickly an intricate structure, called *foliation*, consisting out of *leaves*, will appear.

Train Tracks

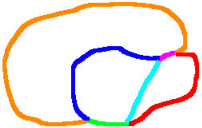
To study foliations and the maps that cause them, we bundle the leaves of a foliation into *branches*. The resulting graph-like object is called a *train track*.



Partition of the foliation into differently coloured regions



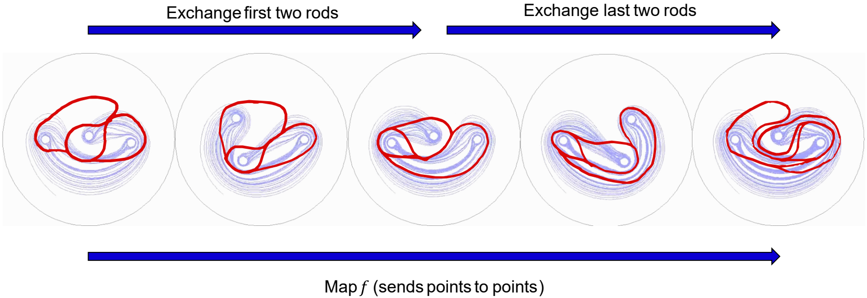
Bundle the regions into one branch



The resulting train track

Train Track mixing

When mixing, the train track behaves similar to the foliation.

$\tau =$  $= f(\tau)$

The last train track $f(\tau)$ looks like a denser version of the first train track τ!

Split

To create new train tracks we can split a branch into two branches and insert a third branch in the middle.

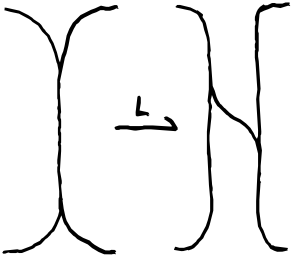


Figure: Left Split of a branch

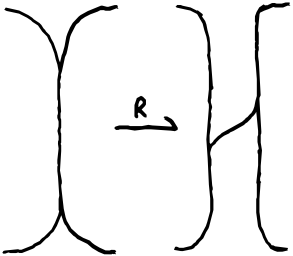
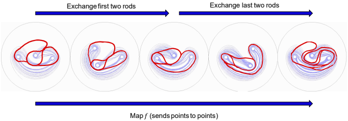


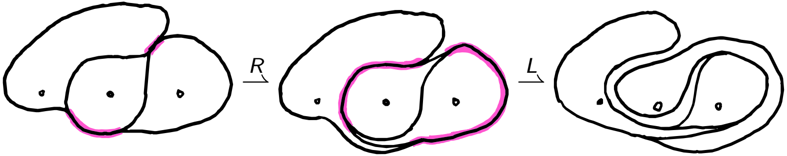
Figure: Right split of a branch

Depending on whether the middle branch goes to the right or to the left, the split is called a left or right split.

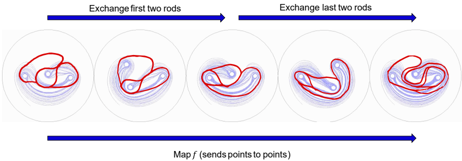
A magic trick

$\tau =$  $= f(\tau)$

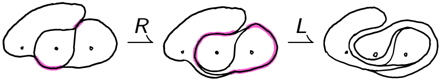
We start with the train track τ which we got from the foliation. I'll apply a special sequence of splits on the highlighted branches.

$\tau =$  $= f(\tau)$

The last train track is exactly the train track $f(\tau)$ from before!

Movement of f on τ: $\tau =$ 

$= f(\tau)$

Splitting train track τ: $\tau =$ 

$= f(\tau)$

Theorem (Agol, 2011)

For a mixing map f, there exists a train track τ and a sequence of splits on τ that reproduces the movement of f on τ.

Thank you for listening!
Danke sehr!
ありがとうございます!

参考文献

Agol, I. (2011). Ideal triangulations of pseudo-Anosov mapping tori. In *Topology and geometry in dimension three. triangulations, invariants, and geometric structures. conference in honor of william jaco's 70th birthday, stillwater, ok, usa, june 4–6, 2010* (pp. 1–17). Providence, RI: American Mathematical Society (AMS). https://doi.org/10.1090/conm/560/11087

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